First-Year First-Semester Exam
August 2018

Answer the following problems and show all your work. Support all statements to the best of your ability. Use a separate sheet of paper for each problem. Each is worth 10 points.

1. Show that for every set X, there is no function $f : X \to 2^X$ such that f is onto.
2. Suppose that $f : X \to Y$ is continuous and that $A \subset X$ is connected. Show that $f(A) \subset Y$ is connected.
3. Let $f, g : X \to Y$ be continuous maps to a Hausdorff space. Prove that the set $\{x \mid f(x) = g(x)\}$ is a closed subset of X.
4. Prove that if X is a compact Hausdorff space, then X is a normal space.
5. State and prove the Contraction Mapping Theorem.

Answer the following with complete definitions or statements or short proofs. Each is worth 5 points.

6. Prove that $\{(a, b] \mid a < b\}$ is a basis for a topology on $\mathbb{R}$.
7. Give the definition of a quotient map.
8. Show that there is no continuous function $f : [0, 1] \to [0, 1)$ which is onto.
9. Does a connected space need to be path connected?
10. State the Intermediate Value Theorem.
11. Describe the one-point compactification of $(0, 1) \cup (2, 3)$.
12. State the Urysohn Lemma.
13. State the Tietze Extension Theorem.
14. Show that if $A \subset \mathbb{R}$ is connected then A is an interval.
15. Is $\mathbb{R}^\omega$ connected in the uniform topology?