2nd Semester Topology Exam
January, 2017

Work the following problems and show all work. Support all statements to the best of your ability. Work each problem on a separate sheet of paper.

1. Show that S^1 is not homeomorphic to S^2.

2. Let X be a connected normal topological space having more than one point. Can X be countable?

3. Prove that homotopy equivalent path connected spaces have isomorphic fundamental groups.

4. Define *retract* and *deformation retract*. Give an example of spaces A and X such that A is a retract of X but it is not a deformation retract of X.

5. Prove that there is no retraction of the disk onto its boundary.

Answer the following with complete definitions or statements or short proofs.

6. Can the set of irrational numbers given the standard metric be presented as a countable union of closed subsets?

7. Define the Stone-Čech compactification and state its characterizing property.

8. Give an example of a connected space X with two points x_0 and x_1 such that $\pi_1(X, x_0)$ is not isomorphic to $\pi_1(X, x_1)$.

9. Compute the fundamental groups $\pi_1(\mathbb{R}^n \setminus 0)$ for all $n > 1$.

10. Is the square map $s : S^1 \to S^1$, $s(z) = z^2$, nulhomotopic ?