1st Year 1st Semester Topology Exam

January, 2017

Work the following problems and show all work. Support all statements to the best of your ability. Work each problem on a separate sheet of paper.

1. Show that the set of functions $f : \mathbb{N} \to \mathbb{Z}$ that are eventually zero is countable. A function $f : \mathbb{N} \to \mathbb{Z}$ is called eventually zero if there is N such that $f(n) = 0$ for all $n \geq N$.

2. Let X be a connected metric space having more than one point. Can X be countable?

3. (a) Show that every continuous map $f : [0, 1] \to [0, 1]$ has a fixed point. Is this true for for continuous maps

(b) $f : (0, 1) \to (0, 1)$?

(c) $f : [0, 1] \to [0, 1)$?

4. (a) Does there exist a continuous surjective map from the 2-sphere S^2 to the interval $(0, 1)$?

(b) Does there exist a continuous injective map from the 2-sphere S^2 to the interval $(0, 1)$?

5. Let $f : X \to X$ be a map of a compact metric space to itself that satisfies the following condition: $d(f(x), f(y)) < d(x, y)$.

(a) Prove that f is continuous.

(b) Show that f has a fixed point and the fixed point is unique.

Answer the following with complete definitions or statements or short proofs.

6. State the Intermediate Value Theorem.

7. State the Cantor-Schroeder-Bernstein Theorem.

8. Is every connected space path connected?

9. What is a basis of a topology? Does the set of all half-open intervals $\{(a, b], [c, d) \mid a < b,\ c < d\}$ form a basis of a topology on $\mathbb{R}$?

10. Is $\mathbb{R}^\omega$ connected in the uniform topology?