MTG5317, FIRST YEAR EXAM, AUGUST, 2016

Part 1. Complete Proofs (7 items, 10 points each). Work each problem on a separate sheet of paper with your name on the sheet. Support all statements to the best of your ability.

1. Suppose that the continuous maps $f, g : X \to Y$ are homotopic and the continuous maps $k, h : Y \to Z$ are homotopic. Prove that $k \circ f$ and $h \circ g$ are homotopic.

2. Prove that in a simply connected space X, any two paths having the same initial and final points are path homotopic.

3. Prove that if X is path connected and x_0 and x_1 are two points of X, then $\pi_1(X, x_0)$ is isomorphic to $\pi_1(X, x_1)$. Recall that $\pi_1(X, x_0)$ denotes the fundamental group of X relative to the base point x_0.

4. Let $p : E \to B$ be a covering map, and let $p(e_0) = b_0$. Let $f : [0, 1] \to B$ be a path beginning at b_0. Prove that f has a lifting $\tilde{f}$ beginning at e_0.

5. Prove that there is no continuous antipode-preserving map $g : S^2 \to S^1$.

6. Prove that if $n \geq 2$, then S^n is simply connected.

7. Prove that the fundamental group of the projective plane P^2 is a group of order 2.

Part 2. Answer the following with complete definitions or statements or short proofs (6 items, 5 points each).

8. Are the spaces S^1 and S^2 homeomorphic?

9. Are the spaces $\mathbb{R}^1$ and $\mathbb{R}^2$ homeomorphic?

10. Complete the following definition: Two spaces X and Y have the same homotopy type if and only if ....

11. Is the identity map $i : S^1 \to S^1$ nulhomotopic?

12. What is the fundamental group of the torus $T = S^1 \times S^1$?

13. Complete the following definition: Let X be a topological space and let A be a subspace of X. We say that A is a retract of X if and only if ....