FIRST-YEAR EXAM - FIRST-SEMESTER TOPOLOGY

Answer all questions and work all problems. Each problem is worth the points allotted.

Problem 1. (10 points) Suppose that $f : X \to Y$ is continuous and onto. Show that if X is compact, then Y is also compact.

Problem 2. (10 points) Suppose that $A \subset X$ is connected. Show that the closure of A, $\overline{A}$, is also connected..

Problem 3. (10 points) State and prove the **Banach Fixed-Point Theorem**.

Problem 4. (10 points) Suppose that X is a compact metric space. Show that if $\{x_i\}$ is a sequence of points in X, then there is a point $z \in X$ and a subsequence, $\{x_{i_j}\}$, such that $x_{i_j} \to z$.

Problem 5. (10 points) Give an example of a sequence of functions $f_i : [0,1] \to [0,1]$ such that $f_i(x) \to g(x)$ for all $x \in [0,1]$, but such that $g : [0,1] \to [0,1]$ is not continuous..

Problem 6. (10 points) Suppose that X is a set and that $f : X \to 2^X$. Show that f cannot be onto.

Problem 7. (10 points) Suppose that X is a complete metric space that is countably infinite. Show that X has an isolated point, i.e., there is a point $x \in X$ such that $\{x\}$ is open in X.

Answer the following with complete definitions or statements or short proofs.

Problem 8. (5 points) State **The Urysohn Lemma**.

Problem 9. (5 points) Show that $[0,1)$ is not compact.

Problem 10. (5 points) State the **Tietze Extension Theorem**.

Problem 11. (5 points) State the **Tychonoff Theorem**.

Problem 12. (5 points) State the **Alexander Subbase Theorem**.

Problem 13. (5 points) Give an example of a Hausdorff space that is not normal.