FIRST-YEAR TOPOLOGY EXAM, JANUARY 2015

NAME: ________________________________

Work the following problems and show all work. Support all statements to the best of your ability. Work each problem on a separate sheet of paper with your name on the sheet.

Problem 1. Show that there is no continuous function $f : [0, 1] \to [0, 1)$ which is onto.

Problem 2. Show that there is no continuous function $f : \mathbb{R} \to \{0, 1\}$ which is onto.

Problem 3. Let X be a metric space and let $f : X \to X$ be a continuous function. Suppose that there is a point $x_0 \in X$ such that $f^n(x_0) \to z$ as $n \to \infty$. Show that $f(z) = z$.

Problem 4. Show that if $A \subset \mathbb{R}$ is connected, then A is an interval.

Problem 5. Prove that if X is compact Hausdorff, then X is a normal space.

Problem 6. State and prove the *Contraction Mapping Theorem.*

Problem 7. Suppose that X is a complete metric space that is countably infinite. Show that X has an isolated point.

Answer the following with complete definitions or statements or short proofs.

Problem 8. State the *Intermediate Value Theorem.*

Problem 9. State the *Urysohn Lemma.*

Problem 10. State the *Tietze Extension Theorem.*

Problem 11. State the *Baire Category Theorem.*

Problem 12. Give an example of a connected space X that is not path connected.

Problem 13. Let $n \geq 2$. Suppose that A is a countable subset of $\mathbb{R}^n$. Let $X = \mathbb{R}^n \setminus A$. Is X arcwise connected?

Problem 14. Suppose that $r > 0$ and that $B_r(x)$ is a ball of radius r centered at x in $\mathbb{R}^n$. Show that $B_r(x)$ is connected.

Problem 15. Show that $[0, 1]$ is uncountable.