MAA 5228 First-Year Exam. May 2016

Do exactly four problems. Work should be presented in a logical fashion in order to receive full credit.

---

1. Let f be a continuous function on $[0,1]$. Prove that f is uniformly continuous.

2. Let $\{G_n\}_{n=1}^{\infty}$ be a family of open sets in $\mathbb{R}^n$ and $\{F_n\}_{n=1}^{\infty}$ a family of closed sets. Prove that $\cup_{n=1}^{\infty} G_n$ is open and $\cup_{n=1}^{N} F_n$ is closed for any finite N. Give an example such that $\cup_{n=1}^{\infty} F_n$ is not closed.

3. Suppose $\sum_{n=1}^{\infty} a_n$ is an absolutely convergent real series. Let b_n be a re-arrangement of a_n. Prove that $\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} a_n$.

4. Let $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, & \text{if} \quad (x,y) \neq (0,0), \\ 0, & \text{if} \quad x = y = 0. \end{cases}$ Prove that f is not continuous at $(0,0)$.

5. Suppose $f'(x) > 0$ for $x \in (0,1)$ and let g be the inverse function of f. Prove that for $x_0 \in (0,1)$, $g'(f(x_0)) = 1/f'(x_0)$.