Answer **four** problems. (If you turn in more, the first four will be graded.) Put your answers in numerical order and circle the numbers of the four problems below your name. Within reason, you may quote theorems as long as you state them clearly.

Name: ______________________________

Problems to be graded: 1 2 3 4 5 6

1. (10 points) Let R be a principal ideal domain. Prove that it satisfies the ascending chain condition on ideals.

2. (a) (2 points) Define *Unique Factorization* Domain.
   (b) (2 points) Define *irreducible element* of an integral domain R.
   (c) (6 points) Prove from your definitions, that if R is a Unique Factorization Domain and p is an irreducible element of R, then the ideal $P = (p)$ generated by p is a prime ideal of R.

3. (a) (3 points) Define *Euclidean* domain.
   (b) (7 points) Prove that every Euclidean domain is a principal ideal domain.

4. Let R be an integral domain and let M be an R-module. Suppose that N is an R-submodule of M. We denote by $\operatorname{Ann}(M)$ the annihilator of M in R, so that $\operatorname{Ann}(M)$ is an ideal of R.
   (a) (4 points) Prove that $\operatorname{Ann}(M/N)\operatorname{Ann}(N) \subseteq \operatorname{Ann}(M)$.
   (b) (3 points) Give an example (with proof) where $\operatorname{Ann}(M/N)\operatorname{Ann}(N) \neq \operatorname{Ann}(M)$.
   (c) (3 points) Give an example (with proof) where $\operatorname{Ann}(M/N)\operatorname{Ann}(N) = \operatorname{Ann}(M)$.

5. (10 points) How many distinct isomorphism types of abelian groups of order $2^4 \cdot 3^4 \cdot 5^4$ and exponent $2^3 \cdot 3 \cdot 5^2$ are there? Justify your answer carefully.

6. (10 points) Let K be a splitting field for $x^4 + 2$ over $\mathbf{Q}$. Calculate the degree of $[K : \mathbf{Q}]$ justifying carefully each step.