Answer **four** problems. (If you turn in more, the first four will be graded.) Put your answers in numerical order and circle the numbers of the four problems below your name. Within reason, you may quote theorems as long as you state them clearly.

Name: ______________________________

Problems to be graded: 1 2 3 4 5 6

1. Give an example of each of the following, with some explanation.
   (a) (3 points) A non-solvable finite group.
   (b) (3 points) A non-cyclic infinite abelian group.
   (c) (4 points) A non-abelian solvable finite group.

2. (10 points) Let G be a finite group and suppose G acts transitively on a set Ω. Prove that $|\Omega|$ (the number of elements in Ω) is finite and that $|\Omega|$ divides $|G|$.

3. (10 points) Let G be a group, and let A and B be normal subgroups of G such that $A \cap B = 1$. Prove that every element of A commutes with every element of B.

4. (10 points) Let G be a group of order 14 and let H be a (multiplicative) cyclic group of order 4. Suppose that $\phi : G \to H$ is a group homomorphism. Let $g \in G$. Prove that there exists some $h \in H$ such that $\phi(g) = h^2$.

5. (10 points) Let $a = (1\ 2\ 3\ 4)(5\ 6)(7\ 8)$ and let $b = (7\ 2\ 8\ 4)(1\ 6)(3\ 5)$ be elements of the symmetric group S_8. Is there an element $\sigma \in S_8$ such that $\sigma a \sigma^{-1} = b$? If so find one.

6. (10 points) Prove that if G is a group of order 56, then G has a normal Sylow 2-subgroup or a normal Sylow 7-subgroup.