Answer **four** problems. (If you turn in more, the first four will be graded.) Put your answers in numerical order and circle the numbers of the four problems below your name. Within reason, you may quote theorems as long as you state them clearly.

Name: ____________________

Problems to be graded: 1 2 3 4 5 6

1. Give an example of each of the following, with some explanation.
   (a) (3 points) A solvable group of order greater than 10.
   (b) (3 points) A non-cyclic abelian group.
   (c) (4 points) A finite group having a non-cyclic abelian proper subgroup.
2. (10 points) Let G be a finite group. Suppose that N is a normal subgroup of even order of G such that the non-trivial elements of N form a single conjugacy class of G. Prove that N is abelian.
3. (10 points) Let $G = A_5$ be the alternating group on 5 letters. Prove that G is a simple group.
4. (10 points) Let $G = D_{2n}$ be the dihedral group of order $2n$. Suppose that $n \geq 4$ is a power of 2. Prove that G is nilpotent and describe the nilpotency class of G.
5. (10 points) Let G be a group of order 30. Prove that G has a normal subgroup of order 15.
6. (10 points) State and prove Sylow's First Theorem.