Department of Mathematics, University of Florida
First Semester Algebra Exam – May, 2015

Answer four problems. If you turn in more than four, only the first four will be graded. Within reason, you may use theorems as long as you state them clearly. When you are done, put the answers in numerical order, put your name in the space below and circle the numbers of the problems you wish graded.

Name:

Problems: 1 2 3 4 5 6

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1. Let $P \subset S_7$ be a cyclic subgroup of order 7. Show that the normalizer N of P has order 42, and find a pair of cycles generating N.

2. Let Q_8 be the quaternion group of order 8. Show that if Q_8 acts faithfully on a set S, then $|S| \geq 8$. Hint: the center lies in every nontrivial subgroup.

3. Let G be a group of order $385 = 5 \cdot 7 \cdot 11$. (i) Show that the 7-Sylow and 11-Sylow subgroups are normal. (ii) Show that the 7-Sylow subgroup is in the center. (iii) Show that G has a cyclic normal subgroup of order 77.

4. (i) Find representatives for all conjugacy classes of elements of order 15 in S_{11}. (ii) Do the same for A_{11}

5. Find (i) representatives of the isomorphism classes of abelian groups of order $504 = 2^3 3^2 7$, and (ii) the number of elements of order 6 in each such group.

6. Suppose G is a finite group, P is a p-Sylow subgroup of G for some prime p and N is a normal subgroup of G. Show that $N \cap P$ is a p-Sylow subgroup of N.