GRADUATE EXAM FOR MTG 5316 INTRO TOPOLOGY I

Work all problems. Use a separate sheet for each problem with your name on the sheet.

Theorems and Proofs. In this section state and prove each theorem. The proofs should be complete without being tedious. Each problem is worth 10 points.

Problem 1. Let X be a metric space and suppose that $A_\lambda \subset X$ is connected for all $\lambda \in \Lambda$. Suppose that there is an $x_0 \in X$ such that $x_0 \in A_\lambda$ for all λ. Show that $\bigcup_{\lambda \in \Lambda} A_\lambda$ is connected.

Problem 2. State and prove the **Contraction Mapping Theorem**.

Problem 3. Let X be a Hausdorff space. Show that if $A \subset X$ is compact, then A is closed.

Problem 4. Let X be a topological space. Suppose that $f : X \to Y$ is continuous. Show that if $A \subset X$ is connected, then $f(A) \subset Y$ is connected.

Problem 5. Suppose that X and Y are metric spaces and that X is compact. Show that if $f : X \to Y$ is continuous, then f is **uniformly continuous**.

Examples and Minor Proofs. In this section, give brief examples, counterexamples, or quick proofs. Each problem is worth 5 points.

Problem 6. State **Urysohn's Lemma**.

Problem 7. Is it possible for there to be a continuous function $f : [0, 1] \to \{0, 1\}$ which is onto?

Problem 8. State the **Bolzano-Weierstrass Theorem**.

Problem 9. Define a continuous function $f : C \to [0, 1]$ which is onto where C is the Cantor set and $[0, 1]$ is the unit interval.

Problem 10. Give an example of a space X which is $\mathbb{T}_1$ but not Hausdorff.

Problem 11. State the **Tietze Extension Theorem**.

Problem 12. Define a **quotient map**.

Problem 13. State the **Baire Category Theorem**.

Problem 14. What does it mean for a topological space X to be **Hausdorff**? **Regular**? **Normal**?

Problem 15. What does it mean for a space X to be **first countable**? Show that every metric space is first-countable.