1st Semester Topology Exam

May 2017

Work the following problems and show all work. Support all statements to the best of your ability. Work each problem on a separate sheet of paper (10 pts each problem).

1. Prove that the circle S^1 is not homeomorphic to the closed 2-disk B^2.

2 . Let $A \subset \mathbb{R}^2$ be an infinite countable subspace.

(a) Can A be connected?

(b) Is $\mathbb{R}^2 \setminus A$ connected?

3. Let $f : X \to \mathbb{R}^2$ be continuous injective map. Is f an embedding if

(a) $X = (0, 1)$?

(b) $X = [0, 1]$?

(c) $X = [0, 1)$?

4. Show that a continuous injective map $f : (0, 1) \to \mathbb{R}$ is an embedding.

5. Show that every compact Hausdorff space is normal.

Answer the following with complete definitions or statements or short proofs (5 pts each problem).

6. Describe all connected subsets of the real line $\mathbb{R}$.

7. State the Contraction Mapping Theorem.

8. Let $\mathbb{R}_\ell$ denote the reals with the lower limit topology, i.e. topology defined by the basis $\{[a, b) \mid a, b \in \mathbb{R}\}$. Is $\mathbb{R}_\ell$

(a) connected?

(b) regular?

9. State the Baire Category Theorem

10. Show that a connected metric space cannot be infinite countable.

11. State the Urysohn Lemma

12. Give definition of a quotient map.

13. Are the spaces $(0, 1) \times [0, 1)$ and $[0, 1) \times [0, 1)$ homeomorphic ?

14. Describe the one-point compactification of $X = (0, 1) \cup (2, 3)$?

15. Let X be a locally connected space. Is every component of X closed ?