Answer **four** problems. (If you turn in more, the first four will be graded.) Put your answers in numerical order and circle the numbers of the four problems below your name. Within reason, you may quote theorems as long as you state them clearly.

Name: ______________________________

Problems to be graded: 1 2 3 4 5 6

1. (10 points) Let p be a prime, let G be a finite group, and let P be a Sylow p-subgroup of G. Let H be a subgroup of G containing the normalizer $\mathrm{N}_G(P)$ of P in G. Prove that then $\mathrm{N}_G(H) = H$.

2. (10 points) State and prove the Second Isomorphism Theorem for groups.

3. Let $n \geq 2$, and let S_n be the symmetric group on n letters.
   (a) (2 points) Define what is a *transposition* of S_n.
   (b) (2 points) Give an example of a transposition of S_n.
   (c) (6 points) Prove that every element of S_n is a product of transpositions of S_n.

4. Let G be a finite group, and assume that G acts transitively on the finite non-empty set Ω. Let $\omega \in \Omega$. We denote by G_ω the set of elements of G that fix ω.
   (a) (3 points) Prove that G_ω is a subgroup of G.
   (b) (7 points) Prove that the number of elements in Ω is exactly $[G : G_\omega]$.

5. Let G be a finite group.
   (a) (5 points) Define what it means to say that G is *solvable*.
   (b) (5 points) Without assuming any result about solvable groups (i.e. just from your definition), prove that if the order of G is 20 then G is solvable.

6. (10 points) Prove or disprove the following statement. Let n be a natural number, let S_n be the symmetric group on n letters, let A_n be the alternating group on n letters, and let $\sigma \in S_n$. Then, if σ has order 2019 then $\sigma \in A_n$.