Answer **four** problems. (If you turn in more, the first four will be graded.) Put your answers in numerical order and circle the numbers of the four problems below your name. Within reason, you may quote theorems as long as you state them clearly.

Name: ____________________

Problems to be graded: 1 2 3 4 5 6

1. (10 points) State and prove Sylow's First Theorem.
2. (10 points) Prove that there is a non-abelian group of order 2019. (Hint: $2019 = 3 * 673$ and 673 is a prime.)
3. S_4 is the symmetric group on four letters.
   (a) (5 points) Prove that every simple subgroup of S_4 is abelian.
   (b) (5 points) Use the result from (a) to prove that if G is a non-abelian finite simple group, then every proper subgroup of G has index at least 5.
4. Let G be a finite group, and assume that G acts on the set Ω. For each $\omega \in \Omega$, we denote by G_ω the set of elements of G that fix ω.
   (a) (2 points) Define what it means to say that the action is *faithful*.
   (b) (2 points) Define what it means to say that the action is *transitive*.
   (c) (6 points) Suppose G acts on Ω faithfully and transitively. Suppose $\nu \in \Omega$ and G_ν is a cyclic group of order p^n, where p is a prime. Prove that there exists some $g \in G$ such that $G_\nu \cap G_{g\nu} = \{1\}$.
5. Let G be a group of order $76 = 4 \cdot 19$.
   (a) (5 points) Prove that G contains a normal Sylow 19-subgroup.
   (b) (5 points) Prove that the center of G contains an element of order 2.
6. (10 points) Let G be a group, and let M and N be normal subgroups of G. Suppose that G/N and N are simple groups. Suppose that $NM \neq G$ and that $N \neq M$. Prove that $M = \{1\}$.