Answer **four** problems. (If you turn in more, the first four will be graded.) Put your answers in numerical order and circle the numbers of the four problems below your name. Within reason, you may quote theorems as long as you state them clearly.

Name: ______________________________

Problems to be graded: 1 2 3 4 5 6

1. (10 points) State and prove Sylow's First Theorem.
2. Consider the symmetric group S_6, and the corresponding alternating group A_6.
   (a) (5 points) How many elements of order 4 are there in S_6? Justify your answer.
   (b) (5 points) How many elements of order 4 are there in A_6? Justify your answer.
3. Let G be the alternating group A_{100}, and assume G acts on a finite set Ω.
   (a) (5 points) Prove that, if $|\Omega| = 90$, then G acts trivially on Ω, i.e. G has 90 orbits of size 1 on Ω.
   (b) (5 points) Prove that if $|\Omega| = 120$ and G does not act trivially on Ω, then G has, at most, 21 orbits on Ω.
4. (10 points) Prove that every group of order 30 has a normal subgroup of order 15.
5. (10 points) Let G be a group, let N be a normal subgroup of G, and let H be a subgroup of G. Assume that H is solvable. Prove that HN/N is a solvable group.
6. (10 points) Let G be a group of order 39. Does G need to be abelian? Prove that your answer is correct.