University of Florida

MAD6407 EXAM August 18, 2016

Name:

ID #:

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Directions: You have 2 hours to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may not use a calculator.

Problem	Possible	Points
1	25	
2	20	
3	15	
4	20	
5	20	
Total	100	

(1) (25 points) The iteration

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n) - \frac{1}{2} f''(x_n) \frac{f(x_n)}{f'(x_n)}}$$

for solving the equation $f(x) = 0$ is known as Halley's method.

(a) Show that the method can, alternatively, be interpreted as applying Newton's method to the equation $g(x) = 0$, with $g(x) = f(x)/\sqrt{f'(x)}$.

(b) Assuming α is a simple root of the equation, and $x_n \to \alpha$ as $n \to \infty$, show that convergence is at least cubic.
Hint: Part (a) might help.

(2) (20 points) Do the following parts which are unrelated.

(a) Derive a Taylor method of order two for the first order initial value problem (IVP)

$$\begin{cases} y' = \dfrac{y^2}{1+t} \\ y(1) = -\dfrac{1}{\ln 2} \end{cases} \tag{1}$$

(b) Derive a numerical method for the following second order IVP by replacing the derivatives with centered differences over the mesh $t_{n+1} = t_n + h$ where $h = 1/N$.

$$\begin{cases} y'' + 2y' + y = \cos t \qquad 0 \le t \le 1 \\ y(0) = 1 \\ y'(0) = 0 \end{cases} \tag{2}$$

(3) (15 points) Consider a quadrature rule of the form

$$\int_0^1 f(x)\,dx \approx Af(0) + Bf'(0) + Cf(\gamma) + Df(1).$$

(a) Determine the constants A, B, C, D, and γ so that the quadrature formula has maximum degree of exactness.

(b) What is the degree of exactness of the formula is part (a)?

(4) (20 points) For the function $f(x) = \ln(x)$ for $x \in [1, 2]$, find the minimax approximation polynomial of degree one. Give the exact value of the minimax error.

(5) (20 points) This problem has the following parts.

(a) Relative to the L^2 norm on the interval $[1, 3]$, find the least squares approximation to the function $f(x) = 1/x$ using polynomials of degree at most one.

(b) Find the least squares error of the approximation above.