Answer **four** problems. (If you turn in more, the first four will be graded.) Put your answers in numerical order and circle the numbers of the four problems below your name. Within reason, you may quote theorems as long as you state them clearly.

Name: ______________________________

Problems to be graded: 1 2 3 4 5 6

1. (10 points) Let G be a cyclic group, and let H be a subgroup of G. Prove that H is cyclic.

2. (10 points) Let G be a finite group, and suppose it is acting on the finite set Ω. Suppose N is a normal subgroup of G and N acts transitively on Ω. Let $\omega \in \Omega$, and let G_ω be the stabilizer in G of ω. Prove that $|G : N| = |G_\omega : G_\omega \cap N|$.

3. Let G be a finite group, let H be a subgroup of G, and let $n = |G : H|$ with $n > 1$.
   (a) (7 points) Prove that if G is simple then G is isomorphic to a subgroup of the symmetric group S_n.
   (b) (3 points) Give an example to show that, under the general assumptions of the question, if G is not simple it is possible that G is not isomorphic to any subgroup of S_n (and prove that your example has these properties).

4. (10 points) Let p be a prime, and let G be a finite group such that each element of G has order a power of p. Prove that, if G is not trivial, then the center $\mathrm{Z}(G)$ of G is not trivial. Deduce that G is nilpotent.

5. (10 points) Let S_n be the symmetric group in $n \geq 1$ letters. Let $\sigma \in S_n$. Describe what is meant by the *cycle type* of σ. Prove that two elements $\sigma, \tau \in S_n$ are conjugate to each other in S_n if and only if they have the same cycle type.

6. (10 points) Prove that a group of order 30 must have a normal subgroup of order 15.