Answer **four** problems. (If you turn in more, the first four will be graded.) Put your answers in numerical order and circle the numbers of the four problems below your name. Within reason, you may quote theorems as long as you state them clearly.

Name: ______________________________

Problems to be graded: 1 2 3 4 5 6

1. Give an example of each of the following.
   (a) (2 points) A non-trivial abelian group.
   (b) (3 points) A non-abelian simple group.
   (c) (2 points) A non-nilpotent solvable group.
   (d) (3 points) A non-abelian nilpotent group.

2. Let G be a group and let $g \in G$.
   (a) (5 points) Define what is meant by the *order* of g.
   (b) (5 points) Let g have infinite order, and suppose that N is a finite normal subgroup of G. Prove, from your definition, that the element $gN \in G/N$ has infinite order.

3. (a) (3 points) Define what it means for a subset of a group G to *generate* G.
   (b) (7 points) Let G be a group, and suppose $|G| = 2^n$. Prove that there is a subset S of cardinality n such that S generates G.

4. (10 points) State and prove the Second Isomorphism Theorem for groups.

5. (a) (5 points) Let S be a simple group and suppose that S acts non-trivially on a finite set A with $|A| = n$. Prove that $|S| \leq n!$.
   (b) (5 points) Suppose $G = A_5$ (the alternating group on five letters) is acting on a set A in such a way that no element of A is fixed by every element of G. Suppose that $|A| = 10$. Prove that G has at most two orbits in its action on A.

6. (10 points) State and prove the First Sylow Theorem.